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A GENERAL MODEL FOR THE STUDY OF THE DYNAMIC BEHAVIOUR OF COHESIVE SEDIMENT BEDS WITH EXTREMELY LARGE DEFORMATIONS

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Final Report for Task D.4 of the COSINUS Project

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by

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1. INTRODUCTION

The success of numerical models for sediment transport and morphodynamics, used as management tools by port authorities and consultants, largely depend on a proper description of the sediment exchange with the bed by erosion and deposition. The modelling of erosion in particular is very difficult because the erosion strength of the bed is highly variable in space and in time. The most detailed models at present include a relatively simple point consolidation model, where the erosion strength is empirically related to the bed surface density. Consequently, only strengthening of the bed is accounted for. However, it is known that extreme forcing by wave induced pore pressure variations within the bed can weaken the structure. The bed may liquefy (structural break-up under shear stresses) and/or fluidise (structural break-up under excess pore pressures) to form a fluid mud layer, which may flow as a gravity current and could cause rapid siltation of navigation channels and harbour docks. Traditional bed models, only suitable for consolidation, have to be replaced by an entirely different model in order to account for fluidisation and liquefaction effects.

Therefore, the aim of this study is the development of a numerical model for the solution of the general dynamic behaviour of a saturated soil under various loading scenarios. The model should be able to solve problems of self-weight consolidation, fluidisation and liquefaction.

Mud beds have a relatively weak, porous structure because it consists of floc-aggregates. Consolidation of mud is a slow process with very large deformations (of the order of 100% for freshly deposited mud). In terms of poro-mechanics, the deformations are extremely large and involve large variations of permeability and other material parameters.

The problem of the fluidisation of a sediment bed by waves has been studied by several researchers. Various experiments have been carried (e.g. Zen & Yamazaki, 1990; de Wit & Kranenburg, 1996). For sand beds, various simple poro-elastic models have been developed (e.g. Yamamoto *et al.*, 1977; Madsen, 1978; Gatmiri, 1990), which all start from the basic poro-mechanics theory developed by Biot (1941, 1955, 1956). Most models are restricted to pure elastic behaviour of the soil skeleton, to small deformations and a constant density and permeability. More recently, models for large deformations, but still constant density, have been proposed (Fowler & Noon, 1999). A general theory is found in chapters 8 and 9 of (Chen & Mizuno, 1990).

In conclusion, existing models do not seem to be suitable for the simulation of the dynamic behaviour of mud beds. The present work has been based on a combination of the generalized Biot theory (Zienkiewicz & Shiomi, 1984; Zienkiewicz *et al.*, 1990) with the solution method for creeping non-Newtonian flows (Crochet *et al.*, 1984) in a mixed-Euler-Lagrangean coordinate system (e.g. Huerta & Liu, 1988).

In a first section the basic equations that describe the bed dynamics are presented. Solution of the bed dynamics equations requires closure relationships for the stresses. In the traditional geotechnical consolidation models empirical stress-density relationships are used (e.g. Toorman, 1999a). These closure equations for the normal stresses are restricted to application to consolidation problems. When fluidisation and liquefaction are involved, another method is required.

2. BASIC EQUATIONS

2.1. Basic assumptions

The sediment-water mixture is simulated as a two-phase medium, where the fluid and solid phase are considered incompressible. This implies that the fluid is considered to be free of gas. The sediment particles form a space-filling structure in which effective stresses can develop. The soil skeleton is completely saturated.

2.2. Reference frame

The computational domain consists of the sediment bed, i.e. a saturated porous medium, the soil matrix filled with pore water. The bed surface is defined as the locus where the effective stress becomes zero. This is a moving boundary. Therefore, the Arbitrary Lagrange-Euler (ALE) formulation (e.g. Huerta & Liu, 1988) is used: the basic equations will be expressed in a mixed Eulerian-Lagrangean reference frame, which allows the general solution of the equations over arbitrarily deforming grids. The grid point is displaced with a velocity:

$$c = \frac{\Delta y}{\Delta t} \qquad \mathbf{(1)}$$

where the displacement Δy can be chosen arbitrarily. In order to keep the relative grid size invariant, the grid point velocity will be calculated as:

$$c(y) = \frac{y\,\Delta h}{h\,\Delta t} \qquad \mathbf{(2)}$$

where h = the instantaneous bed thickness, Δh = the bed surface displacement (at the same horizontal location). This choice avoids unwanted mesh distortions.

2.3. Sediment mass conservation

The sediment mass balance equation is obtained from (Toorman, 1996), rewritten in the ALE form:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\partial \rho}{\partial t} + (U - c)\frac{\partial \rho}{\partial y} = \frac{\partial}{\partial y}\big((U - \dot{u})\Delta\rho\big) \qquad \mathbf{(3)}$$

where: ρ the soil (i.e. the sediment-water mixture) density, $\Delta\rho = \rho - \rho_w$, the excess density, which is proportional to the solids volume fraction $\varphi = \Delta\rho/\Delta\rho_s$, with ρ_s the solids density and ρ_w the water density, $\dot{u}$ is the velocity of the soil skeleton, and U the average mixture velocity, defined as:

$$U = \varphi\,\dot{u} + n\,v_w \qquad \mathbf{(4)}$$

with v_w the pore water velocity and $n = 1 - \varphi$ is the porosity. Since both the fluid and solid phase are considered incompressible: div $U = 0$ (Toorman, 1996). It can be rewritten as:

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial y}\left((\dot{u} - c)\varphi\right) + \varphi \frac{\partial c}{\partial y} = 0 \tag{5}$$

The equation is rearranged in this way because the second term in a weak finite element (FE) form allows setting the surface boundary value, which conveniently is 0 in the case that there is no exchange with the water column.

The general volumetric strain-concentration relationship is given by:

$$\mathrm{d}\varepsilon = \frac{\mathrm{d}V}{V} = -\frac{\mathrm{d}\varphi}{\varphi} \tag{6}$$

where: $\varepsilon = \operatorname{div} u$ and V is a infinitesimal volume. Applied to eq.(5) this yields:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{\partial \varepsilon}{\partial t} + \left(\dot{u} - c\right)\frac{\partial \varepsilon}{\partial y} = \frac{\partial \dot{u}}{\partial y} \tag{7}$$

This implies that the strain rate is equivalent to the divergence of the solids velocity. This is also referred to as *the kinematic relation between fluid inflow and storage* (Zienkiewicz, 1982).

2.4. Momentum conservation

The general momentum equation can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho\dot{u}) + \frac{\partial}{\partial y}\left(\sigma' + \tau - \alpha p\right) = -\rho\, g \tag{8}$$

where: σ' = the normal effective stress, τ = the shear effective stress, p = the pore water pressure and g = the gravity acceleration constant. As only slow variations are considered, the time derivative of the velocity is neglected, as in creeping flow calculations (Crochet *et al*, 1984). For similar reasons the term with the total derivative of the density is neglected. The momentum equation then becomes:

$$\rho\left(\dot{u} - c\right)\frac{\partial \dot{u}}{\partial y} + \frac{\partial}{\partial y}\left(\sigma' + \tau - \alpha p\right) = -\rho\, g \tag{9}$$

2.5. Pore water continuity

The pore water flow in one direction is assumed to follow the semi-empirical Darcy-Gersevanov law, e.g. for the vertical direction:

$$-\frac{k}{\rho_w g}\frac{\partial p_e}{\partial y} = n\left(v_w - \dot{u}\right) \tag{10}$$

with $p_e = p - \rho_w\, g\, (h\text{-}y)$, the excess pore pressure, k = the permeability, ρ_w = the density of the pore water, n = the porosity, $\dot{u}_w$ = the average pore water flow (or seepage) velocity. Assuming no bottom drainage, i.e. $U = 0$, eq.(10) can be rewritten with (4) as:

$$\frac{k}{\rho_w g}\,\frac{\partial p_e}{\partial y} = \dot{u} \tag{11}$$

Taking the divergence of (11), and using (7), yields:

$$\frac{\partial \dot{u}}{\partial y} = \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{\partial \varepsilon}{\partial t} + (\dot{u} - c)\,\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{\rho_w g}\,\frac{\partial p_e}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{k}{\rho_w g}\,\frac{\partial p}{\partial y} - k\right) \tag{12}$$

This allows generalizing the pore water continuity to a general 3D case.

3. TRADITIONAL CONSOLIDATION MODELLING

The previously defined basic equations also lie at the basis of traditional consolidation theory. A comprehensive review is presented by Schiffman *et al.* (1985). A summary is presented here in order to show some of the short-comings.

3.1. Non-linear finite strain consolidation theory

For one-dimensional consolidation, only normal stresses occur, i.e. inertia and shear stresses are not considered. In that case the momentum equation (9) reduces to:

$$\frac{\partial}{\partial y}\left(\sigma' + \alpha p\right) = -\rho\, g \tag{13}$$

or:

$$\sigma' + \alpha p = \sigma = -\int_z^h \rho\, g\, \mathrm{d}y + \sigma_h \tag{14}$$

where σ = the total stress and σ_h a possible overburden stress on the bed surface. The sign of the pore water pressure has been inversed, because in consolidation theory one uses the sign convention of positive pore water pressures. In the case of self-weight consolidation $\sigma_h = 0$. Furthermore, we will consider incompressible pore water, which implies that $\alpha = 1$. Equation (14) can be written as:

$$p = p_0 + p_e = \sigma - \sigma' \tag{15}$$

where p_0 = the hydrostatic pressure, and p_e = the excess pore water pressure.

Substitution of (15) into the pore water continuity equation (10) yields:

$$\frac{1}{g}\frac{\partial \sigma'}{\partial y} = \Delta\rho + \rho_w (1-\varphi)\frac{v_w - \dot{u}}{k} \tag{16}$$

where $\dot{u} = v_s$ = the solids velocity. Considering continuity, eq.(4), and no drainage ($U = 0$), (16) can be rewritten as:

$$\frac{1}{g}\frac{\partial \sigma'}{\partial y} = \Delta\rho - \rho_w \frac{\dot{u}}{k} \tag{17}$$

The solids velocity can than be eliminated between (17) and the mass conservation equation, (3):

$$\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial y}\left(k\frac{\Delta\rho_s}{\rho_w}\varphi^2 + k\frac{\varphi}{\rho_w g}\frac{\partial \sigma'}{\partial y}\right) \tag{18}$$

This is the Eulerian form of the traditional finite-strain consolidation equation, which usually is expressed in a material coordinate frame, i.e.:

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial \zeta}\left(\frac{k}{(1+e)\rho_w g} \left(\Delta\rho_s + \frac{\partial \sigma'}{\partial \zeta} \right) \right) \quad \textbf{(19)}$$

were $e = \varphi^{-1} - 1$ = the void ratio, and ζ = the "reduced" material coordinate, which is related to the Eulerian coordinate y by the relationship $\partial\zeta/\partial y = \varphi$. Introduction of the soil compressibility, which is defined as:

$$a_v = \left(\frac{\partial \sigma'}{\partial e} \right)^{-1} \quad \textbf{(20)}$$

and assuming that k and σ' to depend on e alone, allows the transformation of eq.(19) into the well-known equation developed by Gibson *et al*. (1967):

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial \zeta}\left(\frac{k}{(1+e)\rho_w g a_v} \frac{\partial e}{\partial \zeta} \right) - \frac{\Delta\rho_s}{\rho_w} \frac{\mathrm{d}}{\mathrm{d}e}\left(\frac{k}{1+e} \right) \frac{\partial e}{\partial \zeta} \quad \textbf{(21)}$$

The more detailed derivation of (18) and the equivalence with the "Gibson" equation can be found in (Toorman, 1996).

Solution of equation (18) or (19) requires closures for the permeability and for the effective stress. In traditional soil mechanics and geotechnical engineering this is done with empirical relationships as a function of density (or void ratio). See (Toorman, 1999a) for a review and discussion on constitutive equations. The latter paper also lists the short-comings of these closures.

3.2. Linear small-strain consolidation theory

Additional assumptions of small deformations leads to the linear small-strain (or "conventional") consolidation theory of Terzaghi (1942). In practice, this implies the assumption of a constant permeability and a constant compressibility, such that eq.(19) reduces to:

$$\frac{\partial e}{\partial t} = c_v \frac{\partial^2 e}{\partial \zeta^2} \quad \textbf{(22)}$$

where c_v = the consolidation coefficient, defined as:

$$c_v = \frac{k}{\rho_w g \, (1+e) \, a_v} \quad \textbf{(23)}$$

Equation (22) can be solved analytically (e.g. Lee & Sills, 1981).

3.3. Justification of the need for another modelling approach

The short-comings in the constitutive equations for effective stress in the traditional consolidation models (Toorman, 1999a) form one of the arguments to abandon this approach and move to a more sophisticated model were stresses are related to the deformation history.

A second reason to adopt a new model is the fact that the large-strain consolidation theory is only applicable to the consolidation behaviour of a sediment bed, which only allows the prediction of the densification of the bed and the strengthening of its soil skeleton structure. However, natural sediment beds are also subjected to forces which may break down the structure. This requires extension of the model to include shear forces, induced by currents, waves on the bed surface or by non-equilibrium stress conditions within the bed, and to include the effect of varying water pressures due to tides and waves, which induce horizontal pore pressure gradients.

4. MUD BED RHEOLOGY

Solution of the bed dynamics equations requires closure relationships for the stresses. In order to evaluate the performance of the model, the complexity of the rheological equations will be stepwise increased. Pure elastic behaviour will be studied first as this is the most simple case. A more realistic behaviour of the soil requires more complex rheological equations.

4.1. Linear elastic

The most simple rheological equation is obtained for the assumption that the soil skeleton behaves as a linear elastic body, following Hooke's law. *Biot* (1941) developed his first theory using the constitutive equation of an ideal isotropic elastic body *in equilibrium* (Hooke model). The components of the normal elastic strains ($e_i = \varepsilon_{ii}$) are given by:

$$e_i^E = \frac{1+\nu}{E}\sigma_i' - \frac{\nu}{E}N\sigma' \tag{24}$$

and the shear strain components by:

$$\gamma_{ij} = \frac{\tau_{ij}}{G} \tag{25}$$

where: $\sigma' = \Sigma\, \sigma_j'/n$ = average effective stress, with n = dimension; E = soil elasticity modulus; G = soil shear modulus; υ = soil Poisson ratio. The three material parameters are related by:

$$G = \frac{E}{2(1+\nu)} \tag{26}$$

leaving only two independent parameters.

The strain caused by the pore water pressure can only be normal, because of the isotropic nature of the fluid pressure, and, for the same reason, should be equal in all directions:

$$\varepsilon_p = -\frac{p}{3K_1} \tag{27}$$

where: K_1 = the bulk modulus of the soil skeleton under isotropic compression. The total strain is the sum of these two contributions. The total strain is the sum of these two strains, given by equations (5) and (8).

The total stresses can be expressed as a function of strain by reordering the strain-stress relationships, giving:

$$\begin{aligned}\sigma_i &= 2G\left(e_i + \frac{\nu}{1-2\nu}\varepsilon\right) - \alpha p\\ \tau_{ij} &= G\gamma_{ij}\end{aligned} \tag{28}$$

where the first two terms correspond to the effective stress (this is the stress-strain relationship for a linear elastic body); ε = the solids skeleton dilation, defined as:

$$\varepsilon = \sum_{j=1}^{n} \frac{\partial u_j}{\partial x_j} \tag{29}$$

Similarly, the "dilation" of the pore water is defined as the divergence of the fluid displacement:

$$\varepsilon_w = \sum_{j=1}^{n} \frac{\partial U_j}{\partial x_j} \tag{30}$$

where: U_i = components of the fluid displacement vector.

For the present application, the pore fluid is assumed to be incompressible (which implies that the pore fluid is assumed not to contain gas, which may be present due to biodegradation of organic matter in natural muds). Hence, $\alpha = 1$.

In this simple case, the stresses can be substituted into the stress balance equations:

$$\rho\,(\dot{u} - c)\frac{\partial \dot{u}}{\partial y} + G\,\nabla^2 u_i + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial x_i} - \alpha\,\frac{\partial p}{\partial x_i} = -\rho\, g\, \delta_{ij} \tag{31}$$

Hence, the elastic problem can be solved in an integrated way, which requires less computational effort as the stresses are eliminated as variables. The results have been compared with those of the decoupled model, where the stress equations are solved separately, showing that both approaches yield the same solution, except for some negligibly small oscillations which are generated.

4.2. Visco-elastic

Any added complexity to the elastic model requires the rheological equations to be solved decoupled. Various visco-plastic models have been defined in the literature, particularly for non-Newtonian fluids (e.g. Crochet, 1992).

For the Maxwell-B model (e.g. Crochet, 1992), the normal stress equation is written as:

$$\frac{\tau_{ii}}{\lambda} + \frac{\partial \tau_{ii}}{\partial t} + \dot{u}_j\,\frac{\partial \tau_{ii}}{\partial x_j} - 2\,\tau_{ij}\,\frac{\partial \dot{u}_i}{\partial x_j} = 2\,G\left(\dot{e}_i + \frac{\nu}{1-2\nu}\,\dot{\varepsilon}\right) \tag{32}$$

and the shear stress equation as:

$$\frac{\tau_{xy}}{\lambda} + \frac{\partial \tau_{xy}}{\partial t} + \dot{u}_j\,\frac{\partial \tau_{ii}}{\partial x_j} - \tau_{xx}\,\frac{\partial \dot{u}_x}{\partial x} - \tau_{yy}\,\frac{\partial \dot{u}_y}{\partial y} = G\left(\frac{\partial \dot{u}_x}{\partial y} + \frac{\partial \dot{u}_y}{\partial x}\right) \tag{33}$$

where: $\lambda = \eta/G$, the relaxation time. The term with $\dot{\varepsilon}$ in the RHS of (16) is missing in (Crochet, 1992), because there incompressible fluids are considered for which $\dot{\varepsilon}$ = div $U = 0$, while a compressible soil matrix is considered in the present study.

Pure elastic implies infinite viscosity. One can expect that the skeleton viscosity should increase with density, and should become infinite when the density equals the grain density, i.e. the following empirical non-linear relation could be proposed:

$$\eta = \frac{\eta_0}{\left(1 - \frac{\Delta\rho}{\Delta\rho_s}\right)^n} \quad \textbf{(34)}$$

When the gel point is reached, the skeleton is about to break up, i.e. at the gel point the shear modulus should be 0. A more comprehensive model should include thixotropic effects.

The initial deformation of a newly forming bed at gel point is the settling velocity multiplied by the time step.

4.3. General plastic

The most commonly used rheological model for soils is the general plastic model. The constitutive relation (incremental form of visco-elastic) can be written as:

$$d\sigma' = 2G\,de_i + \frac{2\nu G}{1-2\nu}d\varepsilon - 2\eta\,d\chi + G\,d\gamma_{ij} \quad \textbf{(35)}$$

with addition of a yield criterion to include plasticity. This model can be extended with a creep model.

5. VALIDATION AND CALIBRATION

Validation actually is equivalent to finding the best rheological description of the material. For soils this is a difficult matter. Validation of the model is possible by testing the performance against experimental data of some well-documented cases, such as consolidation column experiments (e.g. Bowden, 1988; Toorman, 1999a) or wave action experiments (e.g. de Wit & Kranenburg, 1993; Foda & Tzang, 1994; Mehta *et al.*, 1995), using various rheological closures of increasing complexity. For saturated cohesive sediment beds a good model is not yet established. Traditional consolidation models which use an empirical stress-void ratio relationship can be shown to be equivalent to pure elastic behaviour. Wave effects on mud beds have been studied thus far only with very simple visco-elastic models, mainly looking at viscous damping and mass transport (e.g. Maa & Mehta, 1990).

Increasing the complexity of a model increases the number of required material parameters. Methodologies to determine some of them can be found in the literature, as well as some data. The following sections do not intend to be comprehensive.

5.1. Permeability

The first parameter to be considered is the permeability of the bed. The permeability decreases with increasing compaction. For the moment the time dependence of the permeability due to release of immobilized interstitial pore water from aggregates will be ignored. The following empirical relationships can be used (Toorman, 1999a):

$$k = \frac{w_0\,\rho_w}{\Delta\rho_s\,\varphi}\,(1-\varphi)^a \qquad \textbf{(36)}$$

based on the popular Richardson-Zaki relationship (with $a \cong 0.5$ for cohesive sediments), or:

$$k = \frac{w_0\,\rho_w}{\Delta\rho_s\,\varphi}\,(1-\varphi)\;\exp(-\varphi/\varphi_1) \qquad \textbf{(37)}$$

5.2. Rheological parameters

The following parameters to be calibrated are those of the rheological model. The shear modulus of mud beds has been measured by various researchers using shear vane tests or shear wave propagation (Williams & Williams, 1989a).

The experimental data can be approximated by a power law, at least over a certain range of the density. However, there are a few constraints. At the gel point the shear modulus is expected to be zero, because below the space-filling density the material has no strength. The following empirical law is proposed, similar to the one proposed for the erosion strength-concentration relationship (Toorman, 1995):

$$G = \alpha_G\,\left(e^{\varphi/\varphi_g - 1} - 1\right)^{n_G} \qquad \textbf{(38)}$$

For example, data for natural mud by Williams & Williams (1989b) can well be fitted with the parameter values $\varphi_g = 0.045$, $\alpha_G = 50$ and $n_G = 1$. Other data seem to fit better with a relation of the form:

$$G = \alpha_G\,\left(\varphi/\varphi_g - 1\right)^{n_G} \qquad \textbf{(39)}$$

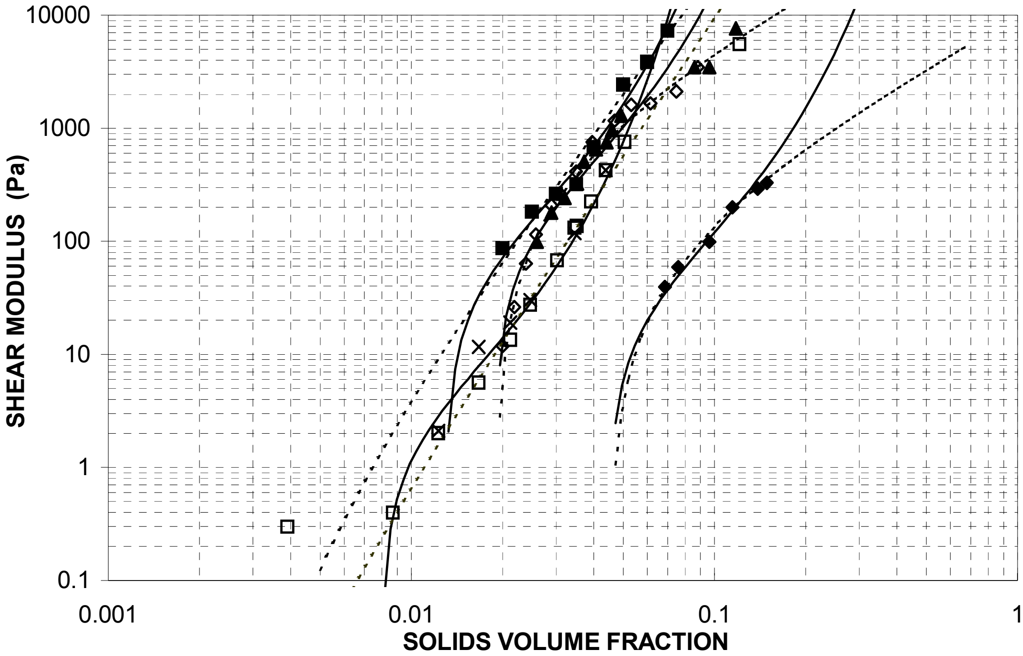

Figure 1**:** Experimental data of shear modulus G and network modulus K for various clay suspensions obtained with various measurement techniques. Bentonite: □ and × = G from vane shear test after 69 and 120 hr recovery time (Alderman *et al.*, 1991). Attapulgite: ◇ = G from pulse shearometer, ▲ = K from equilibrium density profile (Buscall, 1982). K-illite: ■ & natural mud: ◆ = G from pulse shearometer (Williams & Williams, 1989). Lines = empirical fits (full line = eq.25, dashed lines = eq.26).

Similarly, one can expect that the skeleton viscosity should increase with density, and should become very high when the density equals the grain density. As various data in the literature show that storage and loss modulus as a function of density are very similar (e.g. Merckelbach, 1999), a similar empirical non-linear relation could be proposed:

$$\eta = \alpha_\eta \left(e^{\varphi/\varphi_g - 1} - 1\right)^{n_\eta} \quad \textbf{(40)}$$

However, this may not hold for the case of oscillatory loading of small amplitude, even at comparatively low frequencies of around 1 cycle per second or less where one might suppose to encounter quasi-static conditions. Laboratory data on oscillatory vane tests by Merckelbach (1999) show that in this case the storage modulus is about one order of magnitude larger than the loss modulus.

Another interesting finding is a systematic dependence on frequency in the results of the oscillatory method. In the frequency range of measurement the distribution of storage and loss modulus versus frequency might be interpreted as the higher-frequency flank of a complete viscoelastic relaxation maximum as in the phenomenological Cole-Cole model (Cole & Cole, 1941). If this is the case and can be verified by further investigation, in particular by measurements at still lower frequencies, the measured pattern would represent a mechanical relaxation that is activated by oscillatory loading. Possible relaxation mechanisms include thixotropic processes and movement of interlayer water in the clay crystallites. In conclusion, it might be said that one should always be aware whether the shear strength data were obtained by applying a rotation or an oscillatory vane test.

The compression modulus for the particulate network can be calculated from the density profile at equilibrium (i.e. when all excess pore pressures have dissipated) as (Buscall, 1982):

$$K(\varphi) = \frac{\mathrm{d}P}{\mathrm{dln}\varphi} \tag{41}$$

where P is the submerged weight of the sediment layer. Theoretically K and G are related as:

$$K = \frac{2\,(1+\nu)}{3\,(1-2\,\nu)}\,G \tag{42}$$

where ν = the Poisson ratio, having a value in the range 0-0.5, for soils usually in the range 0.25-0.4. Identity is found for ν = 1/8. For the experiments on attapulgite by Buscall (1982) K is found to be identical, within the band of errors, to the dynamic shear modulus (figure 1).

6. FINITE ELEMENT FORMULATION

The equations are solved using a mixed finite element method, i.e. second order interpolation are used for all variables (displacements, stresses and excess densities), except pore pressure for which linear interpolation function are used, in order to avoid spurious oscillations due to not fulfilling certain stability conditions (Crochet *et al.*, 1984). In order to account for the stress history, the model's memory is obtained by implementing an incremental form of the FEM formulation.

The equations are solved subsequently in three groups, first the displacements and pore pressure, then the stresses and, finally, the densities. Since the equations are coupled, an iterative procedure is invoked. A simple first order implicit time stepping scheme is implemented.

As the stress balance equation is a pure advection equation, additional stability is obtained by implementation of self-eliminating artificial diffusion. This is not sufficient when stresses become small. Other stabilisation techniques are still under investigation. Possibly, the method in Crochet *et al.* (1984), where the stress is decomposed artificially, may improve the model's performance, as it introduces permanent diffusion into the equation without generating artificial numerical diffusion.

6.1. Incremental formulation

In order to account for the stress memory, the incremental form (e.g. Smith & Griffiths, 1998) is implemented. The FE form of this set of equations can be written in matrix form as:

$$
\begin{aligned}
K u_{n+1} - Q p_{n+1} &= f^{u}{}_{n+1} \\
Q^{T} u_{n+1} + H \Delta t\, p_{n+1} &= f^{p}{}_{n+1} \Delta t
\end{aligned}
\qquad \textbf{(43)}
$$

Introducing the notation $P(u_{n+1},p_{n+1}) = Ku_{n+1}$ and $R(u_{n+1},p_{n+1}) = Hp_{n+1}$, a Taylor series expansion is done around the value at the previous time step:

$$
\begin{aligned}
P(u_{n+1},p_{n+1}) &= P(u_n,p_n) + K_T \Delta u_n + L_T \Delta p_n \\
R(u_{n+1},p_{n+1}) &= R(u_n,p_n) + J_T \Delta u_n + H_T \Delta p_n
\end{aligned}
\qquad \textbf{(44)}
$$

where the subscript T refers to a tangent. Substitution yields:

$$
\begin{aligned}
K_T \Delta u_n + (L_T - Q)\, \Delta p_n &= f^{u}{}_{n+1} - P(u_n) + Q p_n \\
(Q^{T} + J_T \Delta t)\, \Delta u_n + H_T \Delta t\, \Delta p_n &= \left(f^{p}{}_{n+1} - R(p_n)\right) \Delta t
\end{aligned}
\qquad \textbf{(45)}
$$

In the case that K and H are independent on u and p (e.g. as in the pure elastic case), this set of equations reduces to:

$$
\begin{aligned}
K \Delta u_n - Q \Delta p_n &= f^{u}{}_{n+1} - K u_n + Q p_n \\
Q^{T} \Delta u_n + H \Delta t\, \Delta p_n &= \left(f^{p}{}_{n+1} - H p_n\right) \Delta t
\end{aligned}
\qquad \textbf{(46)}
$$

6.2. Explicit FE formulation

The explicit incremental FE formulation for the momentum conservation then is:

$$\int N\ \rho\,(\dot{u}-c)\frac{\partial \Delta\dot{u}}{\partial y}\mathrm{d}\Omega\ -\ \int \frac{\partial N}{\partial y}\ \big(\sigma'(\Delta u)+\tau(\Delta u)-\alpha\Delta p\big)\mathrm{d}\Omega\ =$$
$$-\int N\rho\,g\ \mathrm{d}\Omega\ +\ \int \frac{\partial N}{\partial y}\ \big(\sigma'(u)+\tau(u)-\alpha p\big)\mathrm{d}\Omega\ -\ \oint N\sigma_s \mathrm{d}\Gamma \tag{47}$$

where the values of u and p in the RHS are taken at the previous time step. σ_s is the surface stress.

For the general 3D pore water continuity, neglecting variations of the permeability:

$$-\int \frac{\partial M}{\partial x_j}\ \Delta\dot{u}_j\ \mathrm{d}\Omega\ +\ \int \frac{\partial M}{\partial x_j}\frac{k}{\rho_w g}\ \frac{\partial \Delta p}{\partial x_j}\ \mathrm{d}\Omega$$
$$=\int \frac{\partial M}{\partial x_j}\ u_j\ \mathrm{d}\Omega\ +\ \int \frac{\partial M}{\partial x_j}\ k\ \mathrm{d}\Omega\ -\ \int \frac{\partial M}{\partial y}\ \frac{k}{\rho_w g}\ \frac{\partial p}{\partial x_j}\ \mathrm{d}\Omega \tag{48}$$

The boundary integral vanishes because of eq.(11).

For the sediment mass conservation:

$$\int N\frac{\partial \Delta\rho}{\partial t}\ \mathrm{d}\Omega\ -\ \int\!\left(\frac{\partial N}{\partial y}\ (\dot{u}-c)\Delta\rho\ -\ N\Delta\rho\,\frac{\partial c}{\partial y}\right)\ \mathrm{d}\Omega\ =\ 0 \tag{49}$$

The RHS is zero as the boundary integral vanishes at the surface.

In all these equations $\dot{u}$ is approximated by $\Delta u/\Delta t$.

6.3. Non-linear permeability

The permeability is generally given by an empirical relationship between k and φ. Hence, the following additional terms appear:

$$J_T\Delta t\Delta u\ =\ \frac{\Delta t}{\rho_w g}\int \frac{\partial M}{\partial y}\ \frac{\partial k}{\partial \varphi}\ \frac{\partial \varphi}{\partial y}\left(\frac{\partial u}{\partial y}\right)^{-1}\frac{\partial p}{\partial y}\ \Delta u\ \ \mathrm{d}\Omega \tag{50}$$

and:

$$H_T\Delta t\Delta p\ =\ \frac{\Delta t}{\rho_w g}\int \frac{\partial M}{\partial y}\ \frac{\partial k}{\partial \varphi}\ \frac{\partial \varphi}{\partial y}\ \Delta p\ \ d\Omega \tag{51}$$

Numerically, this formulation does not work properly.

Another possibility might be using (4):

$$\frac{\partial k}{\partial u}\ =\ \frac{\mathrm{d}k}{\mathrm{d}\varphi}\ \frac{\mathrm{d}\varphi}{\mathrm{d}\varepsilon}\ \frac{\partial \varepsilon}{\partial u}\ =\ -\varphi\ \frac{\mathrm{d}k}{\mathrm{d}\varphi}\ \frac{\partial \varepsilon}{\partial u}\ =\ 0 \tag{52}$$

Since ε is independent on p, also $\partial k/\partial p = 0$. This would imply that no additional terms appear! The latter formulation has been implemented, but it is unclear which is correct.

6.4. Boundary conditions

At the rigid bottom zero displacements are given as essential (or Dirichlet) boundary conditions for the momentum equations. For the 1D cases, as in the example below, horizontal gradients and displacements are set zero.

At the bed surface the stress and pore pressure history has to be given. Surface pore pressures are given as Dirichlet (or essential) conditions for the pore pressure equation. Pressure and stresses are given on all boundaries, except the bottom, as Neumann conditions for the stress balance (momentum equation). Sediment exchange due to erosion or deposition with the water column above the bed is possible. An additional decrease or rise of the bed surface has to be added to the self-weight settlement to account for the eroded or deposited sediment respectively.

At the bottom no fluxes of sediment or pore water (undrained conditions) are considered, i.e. the layer below the modelled bed is assumed to be in equilibrium or solid. Extension to drained conditions is also possible by imposing the pore water flux as natural boundary condition, but requires modification of the pore water continuity equation (i.e. $U \neq 0$).

6.5. Initial conditions

A practical problem is the necessity to know the complete initial state of the sediment bed. In principle, values of all variables should be known in each node. Often in practice, this is not the case. There are only two realistic situations for which the necessary information can be obtained.

The first case is that of a consolidated bed in equilibrium, i.e. all excess pore pressures are zero, as well as all displacements, and the effective stresses equal the submerged weight of the sediment above. This situation can be obtained by the simulation of self-weight consolidation of an initially homogeneous slurry, as in the example below.

The second case is that of no bed. Here the formation of a new bed during deposition can be simulated by allowing a sediment flux at the bed surface. An assumption has to be made on the density of fresh deposit which forms a new top layer. Based on the knowledge that a soil skeleton requires the contact between particles to form a space-filling structure, it is assumed that the floc-particles are arranged according to a certain maximum volume fraction φ_{max} , which is chosen to equal the maximum packing of equivalent spheres (i.e. 64%). This requires the knowledge of the floc density. Hence, the bed model requires also a flocculation model which allows the computation of the density of the flocs which form the bed surface. The top layer growth of a newly forming bed at gel point is then (Toorman, 2000):

$$\Delta h_D = w_s \frac{\varphi_e}{\varphi_{max}} \Delta t \tag{53}$$

where φ_e = the effective volume fraction of the depositing flocs, including their interstitial pore water = $\varphi_f C/\rho_s$, with $\varphi_f = (\rho_f - \rho_w)/(\rho_s - \rho_w)$ the floc volume fraction and ρ_f the floc density. Due to strong shear forces in the boundary layer, the floc density is expected to be higher than higher up in the water column.

7. RESULTS

As the model is still under development, only preliminary results on consolidation can be presented at this stage. Other applications failed thus far.

7.1. Consolidation

The consolidation of an initially homogeneous sediment slurry with initial density above the space-filling density has been simulated. The initial conditions are representative for typical consolidation column tests, as performed in laboratories. The model parameters are: initial height H_0 = 1 m, initial slurry density ρ_0 = 1110 kg/m3, G = 1000 Pa. Permeability calculated with eq.(36) with w_0 = 3.1 mm/s and a = 5 (as for non-cohesive sediment). Figure 2 shows the settlement curves for various values of the viscosity η. A value of 100 Pa.s yields results of the expected order of magnitude for cohesive sediment slurries. The evolution of the density, excess pore pressure and effective stress are shown in figure 3.

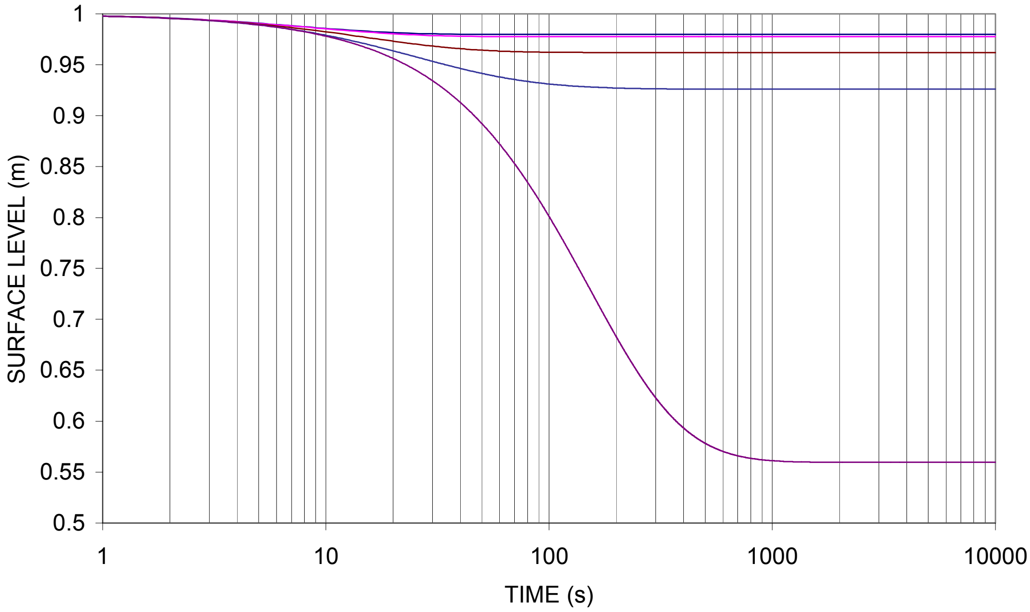

Figure 2: Settlement curves (bed surface as a function of time) for an initially homogeneous slurry: effect of the value of the soil viscosity (from top to bottom) $\eta = 10^6$, 10^5, 10^4, 10^3 and 10^2 Pa.s (other model parameters: see text).

An important shortcoming in these results compared to reality is the lack of surface densification. The model results, i.e. an unchanging surface density, are logic since the model does not allow compaction without loading. What happens in reality probably is the slow dewatering of the surface flocs under the self-weight of the aggregate particles, releasing at least part of the immobilized floc pore water. It may be possible to model this by including some sort of creep. This needs further study, as this requires a distinction between mobile and immobile (i.e. floc) pore water.

The effective stress can then be plotted as a function of the density (figure 3d). It can be seen that the relationship is not unique. In particular, the bending off near the bottom (i.e. at the highest stresses) moves as a function of time, and is in correspondence with experimental data (e.g. compare with fig.4.1 of Merckelbach, 1999).

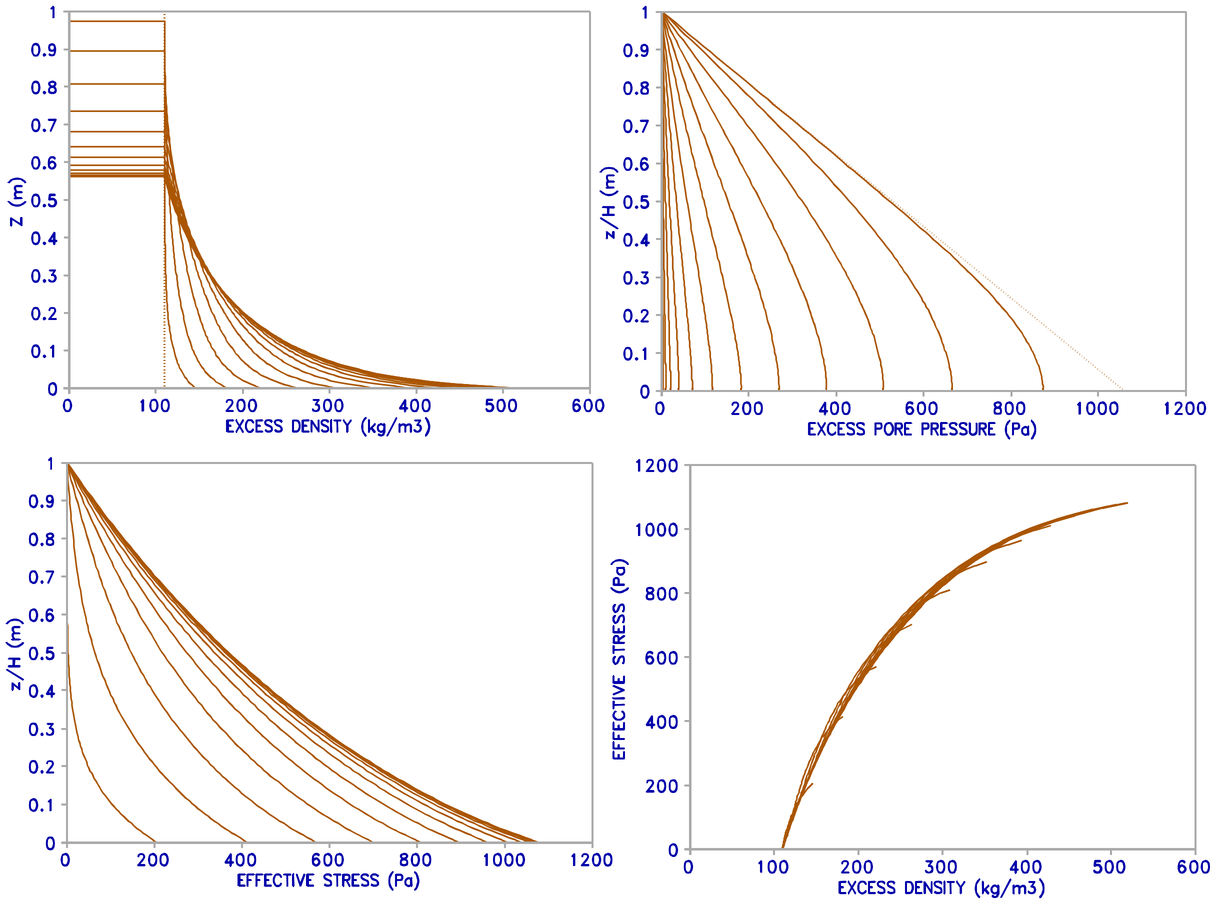

Figure 3: Simulated time evolution of the density (top left), excess pore pressure (top right) and effective stress (bottom left) for the consolidation of a visco-elastic soil skeleton with $\rho_0 = 1110$ kg/m^3, $G = 1000$ Pa and $\eta = 100$ Pa.s. Bottom right: Corresponding effective stress versus excess density.

Attempts to run the model with varying rheological parameters to simulate consolidation failed due to stability problems at very low stress levels near the gel point.

7.2. Fluidisation

Fluidisation of saturated poro-elastic sea-beds has been studied with the traditional Biot theory by many investigators (e.g. Yamamoto *et al.*, 1977; Madsen, 1978; Gatmiri, 1990). This approach assumes negligible deformations, constant density, constant permeability and linear elastic behaviour. Therefore, the applicability is limited to idealised cases of sand beds. It is completely inadequate for the study of dynamic behaviour of mud beds.

However, attempts to simulate fluidisation of an equilibrium layer, obtained after consolidation, due to an oscillating pressure field, with the present visco-elastic model were unsuccessful due to numerical instabilities. Further work is required to make the model more robust.

8. SURFACE DENSIFICATION AND CREEP

One of the short-comings noticed in the model is its inability to predict densification of the bed surface. Since this densification occurs at a constant, i.e. zero, effective stress, it is by definition a creep phenomenon. However, it is not so obvious why the density increases if there is no load at all. A possible explanation may be sought in the small-scale consolidation of the aggregates at the bed surface under their own weight, thereby releasing pore water which is caught within the aggregate due to the hydrophilic properties of the clay particles.

It is hypothesised that the same phenomenon lies at the basis of the problem that a one-to-one relationship between effective stress and solids concentration always leads to theoretical inconsistencies, and that it also helps to explain the non-uniqueness between permeability and density (Toorman, 1996 & 1999a). Particularly for cohesive sediments, one observes that the equilibrium density profile is not homogeneous, i.e. the density increases with depth, whereas it is constant for monodisperse sand. This can only be explained in terms of immobilised pore water, as the actual particles, from which the bed is formed, are aggregates including their immobilised pore water. The deeper in the bed, the more the aggregates are loaded under the submerged weight of the soil matrix above, increasing the release of pore water which is captured within the aggregates. Therefore, one has to redefine balances in terms of three constituents: solids particles, mobile and immobilised pore water (Toorman, 1999b).

8.1. Basic concepts and equations

The mud bed is considered to consist of a packing of aggregate particles which consist of solid particles and immobilized pore water. Hence, not two, but three constituents must be considered. The basic equations are the following. Volume conservation:

$$\varphi_s = 1 - \varphi_w = 1 - (\varphi_m + \varphi_i) \tag{54}$$

with the subscripts: s = sediment, w = water, m = mobile pore water, i = immobilized pore water. The mobile pore water volume fraction is redefined as the porosity n and the immobile pore water volume fraction as the aggregate porosity n_a. The aggregate volumetric concentration thus is $\varphi_a = \varphi_s + n_a$. Mass conservation of the solids:

$$\frac{\partial \varphi_s}{\partial t} + \frac{\partial}{\partial z}\left(v_s \varphi_s\right) = 0 \tag{55}$$

with v_s = the actual settling rate of the skeleton. Mass conservation of the total pore water:

$$\frac{\partial \varphi_w}{\partial t} + \frac{\partial}{\partial z}\left(v_w \varphi_w\right) = \frac{\partial (n + n_a)}{\partial t} + \frac{\partial}{\partial z}\left(v_m n + v_s n_a\right) = 0 \tag{56}$$

with v_m is the actual pore water flow. The immobilized pore water is the same as that of the skeleton. The global pore water velocity then is:

$$v_w = \frac{v_m n + v_s n_a}{n + n_a} \tag{57}$$

Flux balance:

$$v_s(\varphi_s + n_a) + v_m n = U \tag{58}$$

Darcy law:

$$-\frac{k}{\gamma_w}\frac{\partial u}{\partial z} = (1 - \varphi_s - n_a)(v_m - v_s) \tag{59}$$

With substitution of eq.(57), the force balance then becomes:

$$\frac{\partial \sigma'}{\partial z} = -\Delta\gamma_s\varphi_s + \gamma_w(1 - \varphi_s - n_a)\frac{v_w - v_s}{k} = -\Delta\gamma_s\varphi_s + \gamma_w\frac{U - v_s}{k} \tag{60}$$

The latter form is identical to that for rigid particles. In the following only the consolidation for undrained conditions ($U = 0$) will be considered. Equations (54), (55), (56) and (60) plus an aggregate model then constitute a closed system of equations for the five unknowns φ_s, n, n_a, v_s and v_w.

8.2. Aggregate model

Following are some preliminary ideas on the development of an aggregate model.

In reality, individual aggregates cannot be considered in a mud bed. But for a conceptual model the definition of an "aggregate", considered as an entity which contains immobilised pore water, is useful. The aggregates should not be considered as discrete elements, as this would make the model too complex. Quantities have to be averaged over macroscopic volumes. Therefore, it may seem best to describe the averaged aggregate deformation by the superposition of elastic and plastic deformation.

The simplest closure would be to assume that the aggregate size is always such that they occupy the maximum volume, i.e.:

$$\varphi_a = n_a + \varphi_s = \varphi_{\max} \tag{61}$$

This condition must certainly be true at equilibrium. But even then spatial variation of $\varphi_{\max}$ may occur due to shape effects.

The aggregate model should describe the release of pore water when the aggregate is compressed and eventually yields under the effective stresses it is subjected to. One could consider release by compression of the aggregate without break-up, characterised by a compressibility (or, inverse, a bulk modulus), and release by break-up, characterised by a yield criterion. Parameters which are expected to control the pore water release are: internal pore pressure, external pore pressure, load (effective stress) and resistance (aggregate permeability)

The compression of the aggregate is assumed to be elastic, but could be extended with a plastic deformation (strain hardening model). The latter is expected to be negligible and will not be considered. The global plastic behaviour of the skeleton then is assumed to be caused solely by the local break-up of aggregates.

Break-up implies that also the number of aggregates will increase with depth. In other words, various levels of aggregation can be considered and the use of a fractal structure may be useful, bearing in mind that the fractal dimension will increase with depth (Toorman, 2000).

Another option could be a structural kinetics model, which would allow the calculation of an average aggregate size. However, this probably needs more empiricism.

One could also consider a composite permeability: water, which is squeezed out of the aggregates, experiences much higher resistance. Squeezing implies pressure build-up within the immobilised pore water. Outflow implies a pressure gradient between the aggregate pore water and the free pore water. Aggregate strength can then be described by a critical aggregate excess pore water pressure which causes the aggregate to “burst”.

Further work needs to be done to develop a suitable aggregate model.

9. CONCLUSIONS

A general framework has been set up to simulate the dynamic behaviour of a mud bed under the action of gravity and of dynamic shear and pressure forces, allowing the study of strength development under various conditions which occur in nature. The model in its present form still suffers from stability problems, particularly at very low stress levels, which are present in fresh deposits with densities just above the gel point. Therefore the next step in the model development will be the implementation of a much more robust solution method.

Nevertheless, the model performs well for the simulation of self-weight consolidation of an initially uniform slurry. The major shortcoming is the failure to reproduce the densification at the bed surface. It seems that this can only be explained by considering explicitly the floc-bound pore water, which is much more immobilized, but partly may be released slowly under the weight of the solid particles. Similarly, when buried deeper, this process will be faster under the submerged weight of the sediment above. This is a creep-like phenomenon, which is not yet accounted for.

In a next step, the model will be applied for other conditions, such as the dynamic behaviour under waves and currents, including 2DV situations.

The present model does not account for multiple grain sizes. This is not so much of importance, because once a bed is formed, the relative location of the various layers will not change, as segregation occurs in the suspension phase. In principle, grain size effects can be included indirectly in the empirical material parameter closures.

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